Capacitors And Dielectrics

In this small e-book we'll learn about capacitors and dielectrics in short and then we'll have some questions discussed along with their solutions. I'll also give you a practices test series which you can solve on your own.

Brief notes

Capacitors And Capacitance

- A capacitor (formerly known as condenser) is a device that can store electronic charge and energy.
- Capacitors are important components used in electronics and telecommunication devices for example radio , television receivers, transmitter circuits etc.
- Capacitor is a device used for storing electronic charge.
- All capacitors consists of two metal plates (or conductors) separated by an insulator (air, vacuum or any other dielectric medium).
- Figure 1 below shows the symbol used to represent a capacitor.

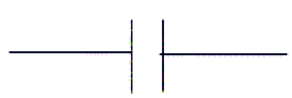


Figure 1

- Capacitor gets charged when a battery is connected to it.

- So when there is a potential difference between two metal plates of the capacitor shown below in the figure.

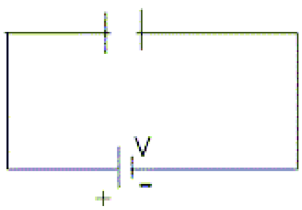


Figure 2

- Capacitor gets discharged on joining two of its plates.

- If V is the potential difference between two plates of the capacitor and q is the amount of charge developed on each plate then q/V is constant for the capacitor since q is proportional to V.

- The ratio of charge on either plate to the potential difference between the plates is called capacitance C of the capacitor. Thus,
  $C=q/V$
  or,
  $q=CV$

- Unit of capacitance is Farads (F) or CV^{-1}.

- 1F is very large unit of capacitance. Practically capacitors with capacitance of the order of micro farads (μF) are used in circuits of radio receivers, transmitters etc. Thus,
  $1\mu F=10^{-6}$ (micro)
  $1nF=10^{-9}$ (nano)

$1pF=10^{-12}$ (pico)

- For any capacitor it's capacitance is constant and depends on shape , size , separation f the two conductors and also on insulating medium being used for making capacitor.

- Capacitance of parallel plate capacitor having vacuum or air acting as dielectric or insulating medium is
  $C= (\varepsilon_0 A)/d$
  where,
  C= capacitance of capacitor
  A= area of conducting plate
  d= distance between plates of the capacitor
  $\varepsilon_0=8.854\times 10^{-12}$ and is known as electric permittivity in vacuum.

- If k is the relative permittivity of the dielectric medium then
  $\varepsilon=\varepsilon_0 k$
  thus capacitance of parallel plate air capacitor in presence of dielectric medium of electric permittivity ε is
  $C=\varepsilon A/d$

- Capacitance of spherical capacitor having radii a, b (b>a) with
  (a) air as dielectric between them
  $C=(4\pi\varepsilon_0 ab)/(b-a)$
  (b) dielectric with relative permittivity ε
  $C=(4\pi\varepsilon ab)/(b-a)$

Series And Parallel Combination Of Capacitors

(A) Parallel combination of capacitors

- Capacitors connected in parallel combination have same potential difference across their terminals. Figure below shows two capacitors connected in parallel between two points A and B

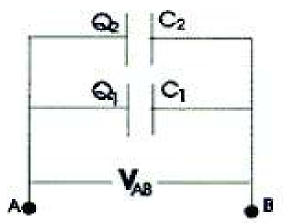


Figure 3

- Right hand side plate of capacitors would be at same common potential V_A. Similarly left hand side plates of capacitors would also be at same common potential V_B.

- Thus in this case potential difference $V_{AB}=V_A-V_B$ would be same for both the capacitors, and charges Q_1 and Q_2 on both the capacitors are not necessarily equal. So,
  $Q_1=C_1V$ and $Q_2=C_2V$

- Thus charge stored is divided amongst both the capacitors in direct proportion to their capacitance.

- Total charge on both the capacitors is,
  $Q=Q_1+Q_2$
  $=V(C_1+C_2)$
  and
  $Q/V=C_1+C_2$

- So system is equivalent to a single capacitor of capacitance
  $C=Q/V$

- When capacitors are connected in parallel their resultant capacitance C is the sum of their individual capacitances.

- The value of equivalent capacitance of system is greater then the greatest individual one.

- If there are number of capacitors connected in parallel then their equivalent capacitance would be
  $C=C_1+C_2+ C_3$..........

- Thus for capacitors connected in parallel combination their resultant capacitance C is the sum of their individual capacitances.

- Also for parallel combination of capacitors their resultant capacitance C is greater then the capacitance of greatest individual one.

(B) Series combination of capacitors

- Figure below shows two capacitors connected in series combination between points A and B.

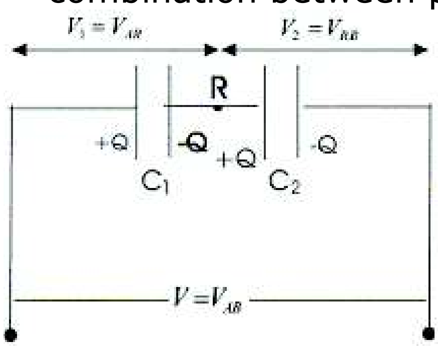


Figure 4

- In series combination of capacitors potential difference across each capacitor is different but charge on each capacitor is same.

- Both the points A and B are maintained at constant potential difference V_{AB}.

- In series combination of capacitors right hand plate of first capacitor is connected to left hand plate of next capacitor and combination may be extended foe any number of capacitors.

- In series combination of capacitors all the capacitors would have same charge.

- Now potential difference across individual capacitors are given by
  $V_{AR}=Q/C_1$
  and,
  $V_{RB}=Q/C_2$

- Sum of V_{AR} and V_{RB} would be equal to applied potential difference V so,
  $V=V_{AB}=V_{AR}+V_{RB}$
  $=Q(1/C_1 + 1/C_2)$

- Thus,
  $1/C=1/C_1+1/C_2$

- Resultant capacitance C of the capacitors connected in series combination is equal to the sum of reciprocals of their individual capacitances.

- Here in case of series combination C is less then the capacitance of smallest individual capacitor.

Energy stored in capacitor

- Energy stored in capacitor is
  $E=QV/2$
  or $E=CV^2/2$
  or $E=Q^2/2C$
  factor 1/2 is due to average potential difference across the capacitor while it is charged.

- Battery supply QV amount of energy during charging a capacitor but energy stored in capacitor is QV/2 , the another half of energy is transferred into the circuit resistance in the form of heat. Thus,
  heat in the wire=energy supplied by battery-energy stored in the capacitor
  $=QV-QV/2 = QV/2$

- If u denotes the energy per unit volume or energy density then
  $u=(1/2)\varepsilon_0E^2$

- The result for above equation is generally valid even for electrostatic field that is not constant in space.

Effect of Dielectric

- Dielectrics are non conducting materials for ex- Glass, mica, wood etc.

- What happened when space between the two plates of the capacitor is filled by a dielectric was first discovered by faraday.

- Faraday discovered that if the space between conductors of the capacitor is occupied by the dielectric, the capacitance of capacitor is increased.

- If the dielectric completely fills the space between the conductors of the capacitor, the capacitance is increased by a factor K which is characteristics of the dielectric and this factor is known as the dielectric constant.

- Dielectric constant of vacuum is unity.

- Consider a capacitor of capacitance C_0 is being charged by the connecting it to a battery.

- If Q_0 is the amount of charged on the capacitor at the end of the charging and V_0 is potential difference across the plates of the capacitor then
  $C_0=Q_0/V_0$
  Thus charge being placed on the capacitor is
  $Q_0=C_0V_0$

- If the battery is disconnected and space between the capacitor is filled by a dielectric the P.D decrease to a new value
  $V=V_0/K$.

- Since the original charge is still on the capacitor, the new capacitance will be
  $C=Q_0/V=KQ_0/V_0=KC_0$

- From above equation it follows that C is greater then C_0.

- Again if the dielectric is inserted while the battery is still connected then battery would have to supply some amount of charge to maintain the Potential Difference between the plates and then total charge on the plates would be $Q=KQ_0$.

- In either of the cases, capacitance of the capacitor is increase by the amount K.

- For a parallel plate capacitor with dielectric of dielectric constant K between its plates its capacitance becomes
  $C=\varepsilon A/D$ where $\varepsilon=K\varepsilon_0$

- When a sufficiently strong electric field is applied to any dielectric material it becomes a conductor and this phenomenon is known as dielectric breakdown.

- The maximum electric field a material can withstand without the occurrence of breakdown is called dielectric strength of that material.

- Thus field across the capacitor should never exceed breakdown limits in order to store charge on capacitor without leaking.

Quick Recap of Formulas

S.No.	Physical Quantity	Formula
1	Capacitance	C=Q/V
2	Capacitance of parallel plate air capacitor	$C= (\varepsilon_0 A)/d$
3	Capacitance of air capacitor in presence of dielectric	$C= (\varepsilon A)/d$
4	Capacitance of spherical capacitor having radii a, b (b>a)	(a) air as dielectric between them **$C=(4\pi\varepsilon_0 ab)/(b-a)$** (b) dielectric with relative permittivity ε **$C=(4\pi\varepsilon ab)/(b-a)$**
5	Capacitance of a spherical capacitor (b>a)	$C = \frac{4\pi\varepsilon_0 ab}{(b-a)}$
6	Capacitance of a cylindrical capacitor (b>a) and l is length of cylinder	$C = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}$
7	Parallel combination of capacitors	$C=C_1+C_2+C_3$..........
8	Series combination of capacitors	$1/C=1/C_1+1/C_2+$...........
9	Energy stored in a capacitor	$U=QV/2$ $U=CV^2/2$ $U=Q^2/2C$
10	Energy density in a capacitor	$u = \frac{\varepsilon_0 E^2}{2}$ where E is electric field in the space between plates

Subjective Type Solved Problems

Question 1: A parallel plate air capacitors has plate area 0.2 m^2 and has separation distance 5.5 mm. Find
(a) Its capacitance when capacitor is charged to a potential difference of 500 volts
(b) Its charge
(c) Energy stored in it
(d) Force of attraction between the plates

Solution:
(a) We know that for a parallel air capacitor capacitance is given by
$C = (\varepsilon_0 A)/d$
Given in the question A = 0.2 m^2 and d = 5.5 mm = .0055 m also we know that $\varepsilon_0 = 8.854 \times 10^{-12}$ C^2/Nm^2.
Thus,
$C = (8.854 \times 10^{-12} \times 0.2)/.0055$
$= 3.231$ nF

(b) We know that charge on the capacitor is given by
Q = CV
Where V is the potential difference between plates of the capacitor which is V = 500 V.
$Q = 3.231 \times 10^{-9} \times 500$
$= 1.615 \times 10^{-6}$ C

(c) Now energy stored in the capacitor is given by
$U = (CV^{2)}/2$
Putting the desired values in the equation we find
$U = (3.231 \times 10^{-9} \times 500 \times 500)/2$
$= 4.04 \times 10^{-7}$ Joule

(d) Now we have to calculate the attractive force between the capacitor plates.

If F is the amount of force and d is the separation distance between the plates then work done by any person to pull the plate must be equal to the increase in energy of the system. Thus,

$W = U = Fd$

Or $F = U/d$

Which is

$F = 4.04 \times 10^{-7} / .0055$

$= 7.34 \times 10^{-5}$N.

Question 2: Consider a system of capacitors where two parallel plate air capacitors each of capacitance C are connected in series to a battery of EMF ξ. Now one of the capacitor is filled uniformly with a dielectric of dielectric constant K. What would happen to electric field strength of that capacitor and what would be the change in electric field strength? Calculate the amount of charge that flows through the battery?

Solution:

First we would have to calculate the charge and voltage on each capacitor. Given that capacitance of both the capacitors is same let it be C. Since both the capacitors are connected in series combination so charge on both the capacitors would be same which lead to same potential difference V across each capacitor which is

$\xi = V + V$ or $V = \xi/2$

Now charge on each capacitor is

$Q = CV = C\xi/2$ in the absence of dielectric.

Now one of the capacitor is being filled up with dielectric of dielectric constant K. So capacitance now becomes KC, equivalent capacitance of the system now becomes

$$\frac{1}{C'} = \frac{1}{KC} + \frac{1}{C}$$

Or,

$$C' = \frac{KC}{1+K}$$

For finding change in electric field strength we'll calculate potential difference across the capacitor filled with the dielectric.

$$V' = \frac{Q'}{KC} = \frac{KC}{2(1+K)}\frac{\xi}{KC} = \frac{1}{2}\left(\frac{\xi}{1+K}\right)$$

Since V is directly proportional to electric field so as V' decreases ½(1+K) times the electric field strength also decreases by the same amount.
Now flow of charge would be
ΔQ = Q'-Q

$$\Delta Q = \frac{KC}{2(1+k)}\xi - \frac{C}{2}\xi = \frac{1}{2}C\xi\frac{(1-K)}{(1+K)}$$

This is the required answer.

Question 3: The spherical capacitor described in has charges + Q and - Q on its inner and outer conductors. Find the electric potential energy stored in the capacitor?

Solution:
In this problem we have to find the energy stored in a capacitor, U. We know that the spherical capacitor has capacitance

$$C = \frac{4\pi\varepsilon_0 ab}{(b-a)} \qquad (1)$$

where a and b are the radii of the inner and outer conducting spheres. For calculating energy stored in capacitor remember the relation

$U=Q^2/2C$ (2)

Now putting the value of C from equation 1 in equation 2 we can find the energy stored in this capacitor which is

$$U = \frac{Q^2}{8\pi\varepsilon_0}\frac{(b-a)}{ab}$$

Question 4:
(a) Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by

$$\left(\mathbf{E}_2 - \mathbf{E}_1\right) \bullet \hat{n} = \frac{\sigma}{\varepsilon_0}$$

Where $\hat{n}$ a unit vector is normal to the surface at a point and σ is the surface charge density at that point. (The direction of $\hat{\mathbf{n}}$ is from side 1 to side 2.)
(b) Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another.

Solution:
(a) We know that statement of Gauss's law in integral form is

$$\oint E.dS = \frac{Q_{enc}}{\varepsilon_0} \qquad \textbf{(1)}$$

Now we have to show that that the electric field always undergoes a discontinuity when surface charge σ is crossed.

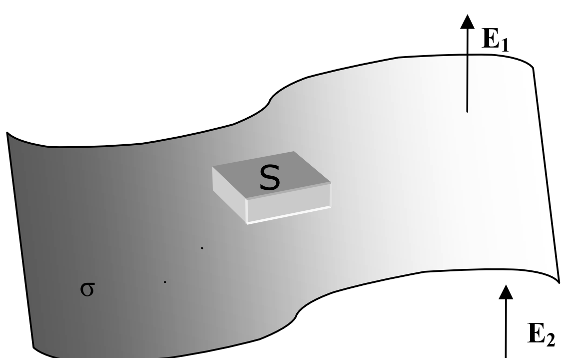


Figure 1

Let's draw a wafer-thin Gaussian pillbox, extending just over the edge in each direction. Apply Gauss's law to Gaussian pillbox S of cross-sectional area A whose two ends are parallel to the interface. The ends of the box can be made arbitrarily close together. In this limit, the flux of the electric field out of the sides of the box is obviously negligible, and the only contribution to the flux comes from the two ends. Thus

$$\oint \mathbf{E.dS} = \left(\mathbf{E_1} - \mathbf{E_2}\right) \bullet \hat{n}A$$

where E_1 is the component of electric field normal to the interface immediately above the surface and E_2 is the component of electric field normal to the interface immediately below the surface The charge enclosed by the pill-box is simply σA, where σ is the sheet charge density on the interface . Thus, Gauss' law yields

$$\left(\mathbf{E_2} - \mathbf{E_1}\right) \bullet \hat{n} = \frac{\sigma}{\varepsilon_0}$$

Where $\hat{n}$ a unit vector is normal to the surface at a point and σ is the surface charge density at that point. (The direction of $\hat{\mathbf{n}}$ is from side 1 to side 2.) Thus that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another *i.e.*, the presence of a charge sheet on an interface causes a discontinuity in the perpendicular component of the electric field.
Just outside a conductor, the electric field is $(\sigma\hat{\mathbf{n}})/\varepsilon_0$.

(b) Now we have to show that tangential component of E, by contrast, is always continuous. For this consider the figure shown below

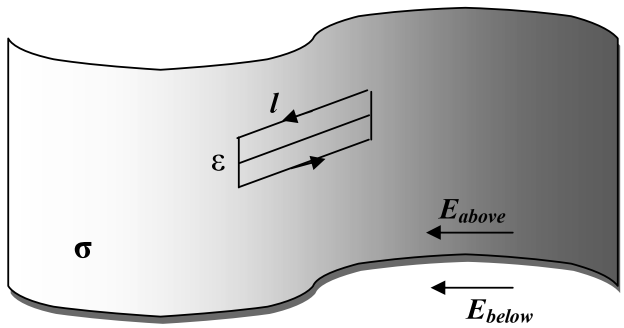


Figure 2

In the above figure consider the thin rectangular loop of length *l* and width $\varepsilon \to 0$.
Now we consider one of the Maxwell's equation known Faraday's law in integral form which is

$$\oint_C \vec{E} \circ d\vec{l} = -\frac{d}{dt}\int_S \vec{B} \circ \hat{n}\, da \quad \text{Faraday's law (integral form)}$$

Here right hand side leads to $A(\mathbf{B})_{normal}$, but A—›0 as sides of the loop are very short nearly approaching zero i.e., $\varepsilon \to 0$, hence contribution due to magnetic effects vanishes. We thus have,

$$\oint \mathrm{E} \bullet \mathrm{dl} = 0$$

Now if we apply this integral to thin rectangular loop as shown in figure 2 then the dominant contribution to the loop integral comes from the long sides because he length of the short sides is assumed to be arbitrarily small which ends up giving nothing. Thus we have

$$E_{above} - E_{below} = 0$$

or,

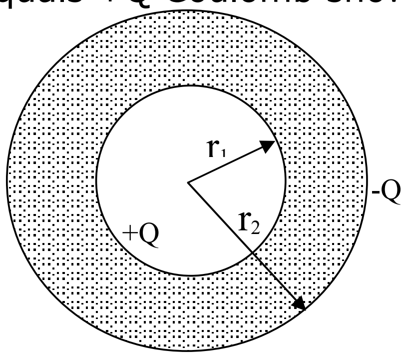
$$E_{above} = E_{below}$$

where, E_{above} and E_{below} are tangential components of electric field thus there can be no discontinuity in the parallel component of the electric field across an interface.

Question 5: Find the capacitance of an isolated spherical conductor of radius r_1 surrounded by an adjacent concentric layer of dielectric with dielectric constant K and outside radius r_2.

Solution:

Let us consider that conductor in the problem has charge equals +Q Coulomb shown below in the figure.



To determine the capacitance we need to find the potential difference between conductor inside the concentric dielectric layer and the region outside the dielectric layer wich is supposed to have charge –Q. Thus,

$$V_+ - V_- = \int_{r_1}^{r_2} \frac{Q}{4\pi\varepsilon_0 K r^2} dr + \int_{r_2}^{\infty} \frac{Q}{4\pi\varepsilon_0 r^2} dr$$

$$V_+ - V_- = \frac{Q}{4\pi\varepsilon_0 K}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{Q}{4\pi\varepsilon_0 r_2}$$

Thus,

$$V_+ - V_- = \frac{Q}{4\pi\varepsilon_0 K}\left(\frac{(K-1)}{r_2} + \frac{1}{r_1}\right)$$

Hence the required capacitance is

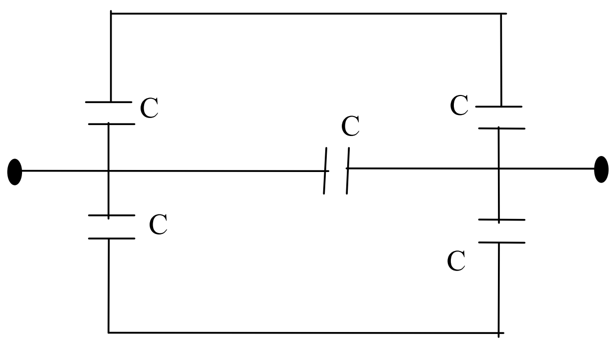
$$C = \frac{Q}{V_+ - V_-} = \frac{Q 4\pi\varepsilon_0 K}{Q\left[\frac{(K-1)}{r_2} - \frac{1}{r_1}\right]} = \frac{4\pi\varepsilon_0 K r_1}{(K-1)\frac{r_1}{r_2} + 1}$$

Objective Type Problems

Question 1: Capacitance of a parallel plate air capacitor depends on

(a) Thickness of conducting plates
(b) Charge on the conducting plates
(c) Area of the conducting plates
(d) Distance of separation between the conducting plates

Question 2: Equivalent capacitance of system of capacitors shown below in the figure is



(a) C/2
(b) 2C
(c) C
(d) None of these

Question 3: A parallel plate air capacitor with no dielectric between the plates is connected to the constant voltage source. How would capacitance and charge change if dielectric of dielectric constant K=2 is inserted between the plates. C_0 and Q_0 are the capacitance and charge of the capacitor before the introduction of the dielectric.

(a) $C=C_0/2$; $Q=2Q_0$
(b) $C=2C_0$; $Q=Q_0/2$
(c) $C=C_0/2$; $Q=Q_0/2$
(d) $C=2C_0$; $Q=2Q_0$

Question 4: A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance 2C is similarly charged to a potential difference 2V. The charging battery is then disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is

(a) Zero
(b) $3CV^2/2$
(c) $25CV^2/6$
(d) $9CV^2/2$

Question 5: A parallel plate air capacitor is connected to a battery. The quantities charge, voltage, electric field, and energy associated with this capacitor are given by Q_0, V_0, E_0 and U_0 respectively. A dielectric slab is now introduced to fill the space between the plates with battery still in connection. The corresponding quantities are now given by Q, V, E and U are related to previous quantities as

(a) $Q > Q_0$
(b) $V > V_0$
(c) $E > E_0$
(d) $U > U_0$

Answers:

1. (a), (c) and (d)
2. (b)
3. (d)
4. (b)
5. (a) and (d)

Solution 1: For parallel plate air capacitor
$C = (\varepsilon_0 A)/d$
Which clearly shows that capacitance does not depend on charge on both the conducting plates. Hence answer is
(a), (c) and (d)

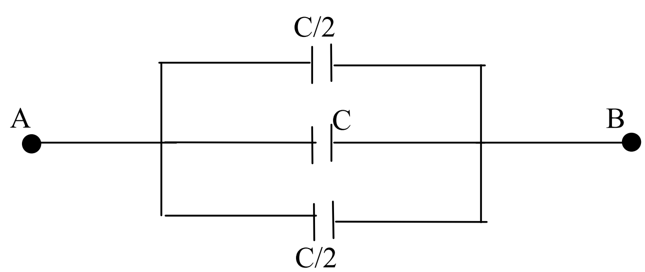
Solution 2: Equivalent capacitance of two capacitors each having capacitance C are connected in series combination

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$$

Or,

$$C' = \frac{C}{2}$$

Now circuit given in question takes the form



All the three capacitors in above figure are connected in parallel combination
Thus,
$C'' = C/2 + C/2 + C = 2C$
This is the required equivalent capacitance.

Solution 3: Given that C_0 and Q_0 are the capacitance and charge of the capacitor before the introduction of the dielectric. Now in the presence of the dielectric capacitance of capacitor becomes
$C = (\varepsilon A)/d$

But we know that relative permittivity of any material is given by
$\varepsilon=K\varepsilon_0$
Where K is the dielectric constant of material.
Thus,
$C = (K\varepsilon_0A)/d$
$= K C_0$
Or, $C= 2 C_0$
Again charge on the capacitor after the introduction of dielectric is given as
$Q=CV$ (a)
And before the introduction of dielectric
$Q_0= C_0V$
Now putting the value of C in equation (a) we find
$Q=2 C_0V$
Or, $Q=2 Q_0$
Where V is same before and after the introduction of dielectric as voltage source used is a constant voltage source.

Solution 4: Charge accumulated on the first capacitor $Q_1=CV$ and charge accumulated on second capacitor is $Q_2= (2C) \times (2V) = 4CV$. Since the capacitors are connected in parallel such that the plates of opposite polarity are connected together, the common potential of the whole system is

$$V' = \frac{Q_2 - Q_1}{C_1 + C_2} = \frac{4CV - CV}{C + 2C} = V$$

Equivalent capacitance $C'=C+2C=3C$. hence the final energy of the configuration is
$U'=C'V'^2/2 = 3CV^2/2$

Solution 5: The potential difference between the plates remains unchanged after the introduction of dielectric because capacitor remains connected to the battery. Thus V remains equal to V_0. Introduction of dielectric increases the capacitance C and hence charge stored in the capacitor also increases since $Q=CV$ thus $Q>Q_0$. Since plate distance and V remains unchanged this implies Electric field E=V/d also remains unchanged. Energy stored in the capacitor increases as it depends on the capacitance C of the capacitor since $U=CV^2/2$. Thus $U>U_0$.